

Optical Nonlinear Dynamics with cold atoms in a cavity

A.Lambrecht, E.Giacobino, J.M.Courty

*Laboratoire Kastler Brossel **

Université Pierre et Marie Curie, case 74, 4 place Jussieu, 75252 Paris Cedex 05, France

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This paper presents the nonlinear dynamics of laser cooled and trapped cesium atoms placed inside an optical cavity and interacting with a probe light beam slightly detuned from the $6S_{1/2}F = 4$ to $6P_{3/2}F = 5$ transition. The system exhibits very strong bistability and instabilities. The origin of the latter is found to be a competition between optical pumping and non-linearities due to saturation of the optical transition.

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I. INTRODUCTION

In an atomic vapor, the interaction of the atoms with one or several near resonant electromagnetic fields is complicated by the fact that the various velocity classes have different detunings from the fields, except when this detuning is very large. Laser cooled atoms in a magneto-optic trap [1] can have velocities as low as a centimeter per second. This means that their Doppler width is smaller than the natural linewidth and that a laser field can be set close to resonance with an atomic transition, all the atoms having the same detuning from the field. In such conditions, the interaction between the atoms and the field is well characterized and one can take advantage of the strong non-linearities of atomic systems while keeping the absorption rather small. In particular, it was shown that a probe beam going through a cloud of cold atoms could experience a strong gain due to Raman transitions involving the trapping beams [2]. When the atoms are placed in a resonant optical cavity without a probe beam, laser action corresponding to that gain feature was demonstrated [3].

When cold atoms interact with a probe laser beam inside an optical cavity, bistability is easily observed at very low input powers (as low as $5\mu\text{W}$) [4]. This bistability effect was observed in the presence of the cooling beams. However, to investigate the nonlinear dynamics of a collection of cold atoms in a cavity in more detail, we needed better controlled conditions, and we studied the behaviour of the system in the absence of cooling beams, right after the trap is turned off. In that case, in addition to bistability, new features were found, such as very pronounced self pulsing oscillations in a wide range of experimental parameters.

While instabilities have been observed in similar conditions in atomic vapors [5], the present situation is much easier to analyse, and we show hereafter that we have been able to find a rather simple model for these instabilities, that invokes a competition between optical pumping and saturation.

The experimental procedure is described in section 2. In section 3 we give a model explaining the dynamical behaviour of the system and compare its results with the measurements.

II. BISTABILITY AND INSTABILITIES WITH COLD ATOMS

In the experiments described in the following, we prepare a cloud of cold cesium atoms in a cell using the background pressure to fill the trap. The trap operates in the standard way [1], with three orthogonal circularly polarized trapping beams generated by a Ti:Sapphire laser and an inhomogeneous magnetic field. The Ti:Sapphire laser is detuned by 2.5Γ (Γ being the linewidth of the upper state) on the low frequency side of the $6S_{1/2}F = 4$ to $6P_{3/2}F = 5$ transition. We obtain a cloud of cesium atoms the typical temperature of which is of the order of 1 mK, which gives a Doppler width much smaller than the natural width. The diameter (2.5 cm) and power (20 mW/cm^2) of our trapping beams allow us to obtain large clouds (about 5 mm in diameter) with densities of the order of $10^{10}\text{ atoms/cm}^3$. The relevant parameter in the experiment is actually the number of trapped atoms in the probe beam, which is measured from the change in the intensity of the probe beam transmitted through the cavity with and without trapped atoms. This number is found to be ranging between 10^7 and 10^8 depending on the pressure of the background gas. As usual, the atoms non resonantly excited into the $6P_{3/2}F = 4$ state and falling back into the $F = 3$ ground state are repumped into the cooling cycle by a laser diode tuned to the $6S_{1/2}F = 3$ to $6P_{3/2}F = 4$ transition.

The cavity is a 25 cm long linear asymmetrical cavity, close to half-confocal, with a waist of $260\mu\text{m}$. Because the cell has optical quality antireflecting windows, we can build a good finesse optical cavity around the atomic cloud (Fig.1). Losses due to the two windows are of the order of 1%. The input mirror has a transmission coefficient of 10%, the end mirror is a highly reflecting mirror. The cavity is in the symmetry plane of the trap, making a 45° angle with the two trapping beams that propagate

*Laboratoire de l'Université Pierre et Marie Curie et de l'Ecole Normale Supérieure, associé au CNRS

in this plane.

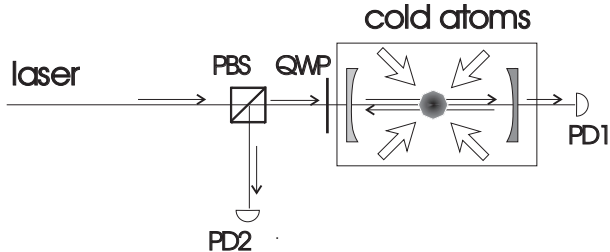


FIG. 1. Experimental set-up showing the cell containing the cold atom cloud in an optical cavity; PBS: polarizing beamsplitter, QWP: quarter wave plate, PD1, PD2: photodiodes; PD1 and PD2 measure the powers respectively transmitted and reflected by the cavity.

To look for bistable behaviour of the optical cavity containing cold atoms, we send a circularly polarized probe beam into the cavity. It can be detuned by 0 to 130 MHz on either side of the $6S_{1/2}F = 4$ to $6P_{3/2}F = 5$ atomic transition.

We measure the power of the beam transmitted through the cavity while scanning the cavity length for a fixed value of the input intensity, as shown in Fig.2, or the input intensity for a fixed value of the detuning. The recording shows the characteristic hysteresis cycle due to bistability, where the output power switches abruptly between low and high values when the length of the cavity is scanned. Switching and hysteresis were also observed when the input power is scanned at fixed cavity length. In some cases overshoot and oscillations in the output power were recorded [4].

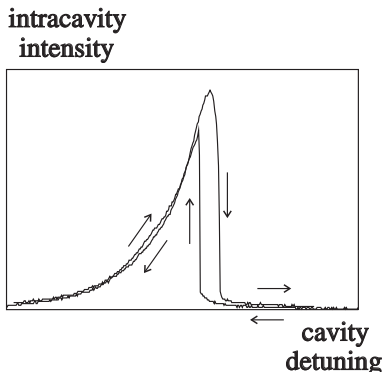


FIG. 2. Recording showing the bistable switching from low to high transmission and back when the cavity length is scanned across the cavity resonance. The trapping laser beams are on. The laser is detuned by 22Γ on the high frequency side of the atomic transition, the input power is $100 \mu\text{W}$.

The shapes of the curves in such a case can be grossly interpreted from the theory of bistability with two-level atoms. However, the fit is approximate and the theory, even including single mode instabilities [6,7], cannot explain the overshoot and oscillations [8]. Actually the transition under investigation in cesium is far from being

a two-level one in the presence of the trap. The fact that standard bistability is observed in most cases to be in fair agreement with two-level atom theory can be explained by the fact that the trapping beams randomize the the ground state population among the various Zeeman sub-levels.

To thoroughly investigate the phenomenon, it was thus indispensable to study the system without the trapping beams. After the trap is loaded, we turn off the trapping laser beams in order to get unperturbed atoms. We have about 20 ms to perform the measurements before most of the atoms have escaped out of the interaction region due to free fall and expansion of the cloud. In the experiment, the bistability parameter C (see definition in section 3) can be as high as 400 just after the atoms have been released.

In a broad range of experimental parameters, we observed bistability and instabilities that exhibit unusual features. Instabilities are present within the whole range of accessible detunings (from -25Γ to 25Γ , where Γ is the linewidth of the excited state, $\Gamma/2\pi = 5.2 \text{ MHz}$), and for input powers ranging from 50 to $300 \mu\text{W}$. These oscillations are somewhat similar to the ones observed in the presence of trapping beams but those were obtained only at high intensity or small atomic detuning. Here, on the contrary, they are observed very easily.

Fig. 3 shows a set of recordings of the output power of the cavity when the length is scanned, for different values of the input power. At very low power (not shown), the curves exhibit neither bistability nor instabilities. Starting for an input power of the order of $30 \mu\text{W}$, oscillations appear on the left hand side of the cavity resonance curve, within some range of cavity detuning. This side is the one on which a bistable switching would occur in a saturated two-level atomic system. At higher input powers, the oscillations disappear and only bistability persists. Let us mention that when the power of the probe beam is too high, the atoms are expelled very rapidly from the beam

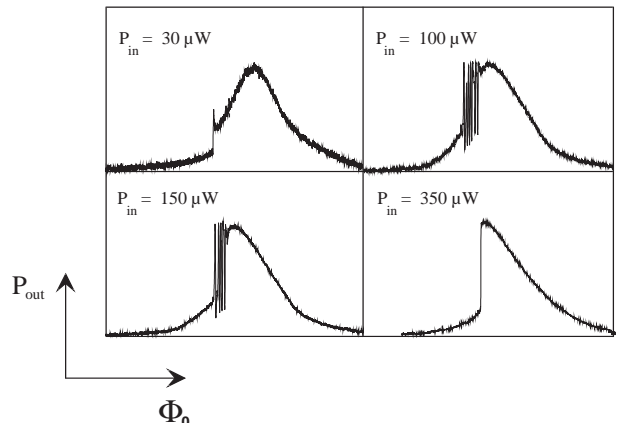


FIG. 3. Recording of the output power P_{out} of the cavity containing cold atoms when the cavity length is scanned, for four different values of the input power P_{in} . The trapping beams are off. The atomic detuning is the same as in Fig. 2.

and only the very early part of the signal can be considered as significant of the nonlinear behaviour of the atoms.

Once enlarged these oscillations look clearly like self-pulsing. Their frequency is comprised between 100 KHz and a few MHz, and they are not due to the scan of the cavity length. To investigate them in more detail, one would want to record them at a fixed cavity length. However, it is not possible to keep the optical cavity length perfectly constant in time because the atoms escape from the original cloud, thus changing the index of refraction. But in such conditions, the effective cavity scan is slow and the oscillations are observed on longer time durations. Such a recording is shown in Fig. 4.

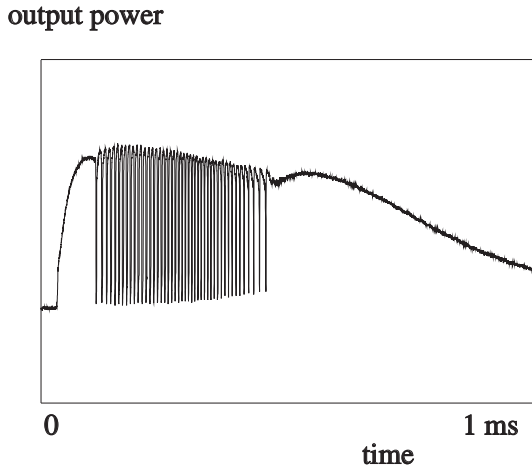


FIG. 4. Recording of the instabilities for a fixed geometrical cavity length. The optical length is slowly scanned by the decrease of the number of atoms. The detuning is the same as in Fig. 2, the input power is $80 \mu\text{W}$, the trapping beams are switched off at $t = 0$.

III. MODEL FOR INSTABILITIES

To understand these oscillations, one has to take into account the hyperfine and Zeeman structure of the considered states. Various optical pumping effects can occur and phenomena linked to it like bistability, multistability [9,10,11,12] and instabilities [5,6,7,8,9,10,11,12,13] have been predicted and observed in alkali vapors. Instabilities necessitate a strong coupling between the atoms and the field, that is a small detuning. Considering that the Doppler width is of the same order as the hyperfine structure in the ground and excited state of cesium, the system is rather intricate to describe, due to the competitive action of the various velocity classes and hyperfine sublevels.

As already pointed out, the situation is much simpler with cold atoms, where one can consider that the field interacts with one hyperfine transition only. Owing to this, we have been able to develop a simple model to understand the origin of the observed oscillations. Roughly, as shown below, they result from the competition between a

fast nonlinear process, the saturation of the optical transition and a much slower one, the optical pumping.

The saturation of the optical transition causes a decrease of the linear index of refraction of the atomic medium when the intensity increases. On the contrary, optical pumping by circularly polarized light is at the origin of a non-linearity that increases the index of refraction with the light intensity: when the atoms are submitted to circularly polarized light, they tend to accumulate in the magnetic sublevels with high m_F number, which have the largest coupling coefficient with the electromagnetic field. Therefore, the two nonlinear processes have opposite effects and compete in our system. The relaxation oscillations are a consequence of the significant difference in the characteristic times of these processes.

Optical pumping tends to empty out the magnetic sublevels of low magnetic number to accumulate all the atoms in the sublevels with highest magnetic number ($m_F = 4$ to $m_F = 5$ transition). Due to the high number of magnetic substates, it takes a rather long time, starting from an equally distributed population, to complete the optical pumping to the highest m_F sublevels of the ground and excited states.

The time evolution of the sum of the populations of these two sublevels for several values of the Rabi frequency of the probe beam (calculated as an average over the hyperfine transitions) is shown in Fig. 5. One can see that even for values close to or larger than the saturation value, the optical pumping rate is much smaller than the natural linewidth. Thus the response time of the nonlinear susceptibility due to optical pumping is much larger than the one due to saturation of the optical transition, which, for a field detuned from resonance, can be considered as being of the order of the detuning.

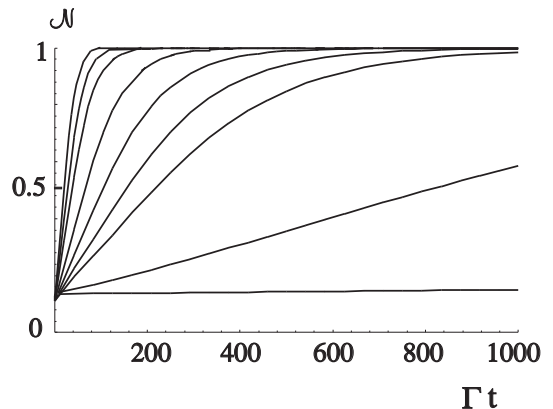


FIG. 5. Optical pumping as a function of time : the curves show the calculated variation of the sum \mathcal{N} of the populations of the $m_F = 4$ sublevel of the ground state and of the $m_F = 5$ sublevel of the excited state as a function of time, starting from a population equally distributed among the ground state sublevels, when the intensity I of the pumping light (defined by Eq. (1)) is equal to (from bottom to top) : 1, 5, 10, 12, 15, 20, 30, 40, 60. Time is in units of Γ . The atomic detuning is 20Γ .

Although the behaviour of the system, involving many variables (all the hyperfine Zeeman populations and coherences and the field), is quite complex, the underlying mechanism can be explained with a simple model, involving basically two differential equations that give the evolution of the intracavity field and that of the atomic orientation in the ground state. To write these equations, we first introduce our basic notations.

The intracavity intensity I of the laser is normalized to the saturation intensity :

$$I = \frac{g^2 |\alpha|^2}{\Gamma^2/4}, \quad (1)$$

where g is the coupling constant of the atoms with the field,

$$g^2 = \frac{d^2 \omega_L}{2\epsilon_0 \hbar S c}, \quad (2)$$

d is the atomic dipole, ω_L the frequency of the probe laser and S the cross section area of the beam. $|\alpha|^2$ is the electric field squared, expressed in units of number of photons per second.

The round trip phase shift Φ_{cav} of the field in the cavity (assumed to be a ring cavity) is the sum of four contributions. First, the phase shift Φ_0 proportional to the geometrical length of the cavity. Second, we have two contributions due to the presence of atoms in the cavity, a linear phase shift,

$$\Phi_L = 2Ng^2/\Gamma\delta \quad (3)$$

and a nonlinear Kerr-like phase shift

$$\Phi_{\text{NL}} = -KI. \quad (4)$$

The nonlinear coefficient K is given by

$$K = 4Ng^2/\Gamma\delta^3, \quad (5)$$

where N is the number of atoms and δ the detuning of the atomic transition frequency ω_0 from the field frequency ω_L , normalized to the atomic transition linewidth $\Gamma/2$

$$\delta = 2(\omega_0 - \omega_L)/\Gamma. \quad (6)$$

Φ_L and Φ_{NL} are the phase shifts corresponding to the presence of two level atoms in the cavity. If we now consider that the ground and excited states have several Zeeman sublevels, the main additional contribution when the atoms interact with circularly polarized light is a term Φ_p coming from the change in the populations of the ground state sublevels and proportional to the ground state orientation p (normalized to 1). Since the square of the Clebsch-Gordan coefficient of the $m_F=4$ to $m_F=5$ transition is about twice the mean square Clebsch-Gordan coefficient for the $F=4$ to $F=5$ transition, Φ_p is equal to Φ_L for $p=1$, that is when the ground state is completely pumped. Thus, we can write

$$\Phi_p = \Phi_L p. \quad (7)$$

The total phase shift in the cavity is then:

$$\Phi_{\text{cav}} = \Phi_0 + \Phi_L + \Phi_{\text{NL}} + p\Phi_L \quad (8)$$

The ground state orientation p increases with the intracavity intensity I at rate βI and decays at a rate γ_p due to magnetic precession in transverse fields and to transitions to other hyperfine sublevels (via non resonant transitions):

$$dp/dt = -\gamma_p p + \beta I(1-p). \quad (9)$$

The pumping rate coefficient β is computed from the calculation presented in Fig. 5 and the relaxation rate γ_p is evaluated from the experimental parameters.

The change of the intracavity field α on a round trip of time duration τ is due to the driving field α_{in} entering through the coupling mirror of transmission t , to the mirror decay coefficient γ_{cav} (with $\gamma_{\text{cav}} = t^2/2$) and to the round trip phase shift Φ_{cav} :

$$\tau d\alpha/dt = t\alpha_{\text{in}} - (\gamma_{\text{cav}} - i\Phi_{\text{cav}})\alpha. \quad (10)$$

The Kerr-like non-linearity has been assumed to have an instantaneous response. In the absence of optical pumping this system is well known to become bistable when the intensity is larger than a threshold intensity I_{bist} given by

$$I_{\text{bist}} = 8\gamma_{\text{cav}}^2/3\sqrt{3}K. \quad (11)$$

In the presence of optical pumping eqs. (8) and (9) have to be solved numerically. They involve two different rates: the optical pumping rate βI and the field evolution rate in the cavity γ_{cav}/τ ($\gamma_{\text{cav}}/2\pi\tau \approx 5$ MHz) which is usually larger than the optical pumping rate, except at very high pump powers.

These equations have been used to calculate the motion of the system. In some range of initial conditions, oscillations and limit cycles are found in the intracavity intensity as well as in the output intensity. To compare these results with the experimental data, we have calculated the output intensity when Φ_0 , i.e. the cavity length, is slowly scanned for several values of the input intensity. The result is shown in Fig. 6. It can be seen that the curves reproduce the experimental recordings in a satisfactory way.

In particular, they show oscillations setting in for intermediate powers. When the input power is strong enough, the oscillations disappear, due to the fact that the optical pumping is fast enough to bring all the atoms in the sublevels of high magnetic number before the oscillations can start. The calculations show that in the unstable region, the ground state orientation may vary by quantities as small as 1%. As a consequence, the period of the instabilities can be much smaller than the typical optical pumping time shown in Fig. 5.

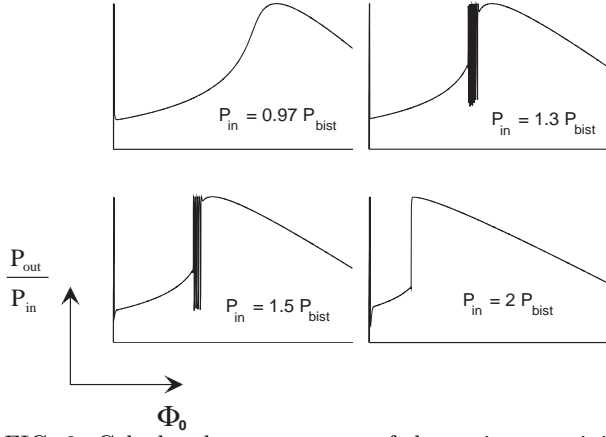


FIG. 6. Calculated output power of the cavity containing cold atoms when the cavity length is scanned for four values of the input power chosen below and above the bistability threshold P_{bist} (P_{bist} is in the absence of optical pumping, cf eq.(10)).

This oscillatory behaviour can be understood by considering that the optical pumping changes the optical length of the cavity. It can then scan the cavity length back and forth in the vicinity of the bistable regime, causing periodic abrupt changes in the cavity transmission as shown in Fig. 7. A similar behaviour was observed in a different system with thermal effects [14].

Instabilities can also be found in the absence of relaxation for the orientation when the cavity length is scanned. This occurs for high intensities and large num-

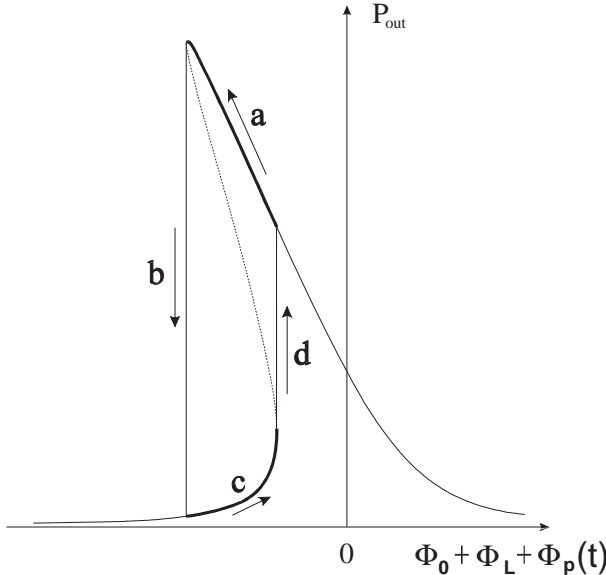


FIG. 7. Diagramme showing the self-pulsing of the bistable system due to optical pumping (for the case of Φ_L and $\Phi_p < 0$); (a) the cavity is in the high transmission state and optical pumping increases the orientation and decreases Φ_L ; (b) abrupt switch towards the low transmission state; (c) the cavity is in the low transmission state, depumping is dominant and the orientation decreases; (d) switch towards the high transmission state.

bers of atoms. In such a case, in the vicinity of a resonance, the optical pumping takes over, brings the cavity first into resonance and then beyond the resonance. After this overshoot, the cavity length scan brings the cavity back into resonance, the optical pumping starts again and so on. Thus the orientation increases by steps, each time the light enters the cavity.

The validity of the model was checked by further experiments. When the atoms are released from the trap, it is possible to optically pump them into the $m_F = 4$ magnetic sublevel of the ground state with an additional circularly polarized beam parallel to the probe beam, but closer to the atomic resonance. This prepumping is done in the presence of a magnetic field directed along the cavity. In such conditions the instabilities disappear. If the magnetic field is absent, the orientation created by the pump field is destroyed by the Larmor precession in transverse magnetic fields and the instabilities persist.

A more complete treatment of the atomic non-linearity and of the optical pumping was also performed, where absorption and saturation of the optical non-linearity were taken into account. In this case, the separation of linear and nonlinear phase shifts is no longer possible. Instead, one defines a total phase shift due to two-level atoms

$$\Phi_1 = \frac{2Ng^2}{\Gamma} \frac{\delta + i}{1 + \delta^2 + 2I} \quad (12)$$

or with the help of the bistability parameter C given by

$$C = g^2 N / \gamma_{\text{cav}} \Gamma, \quad (13)$$

$$\Phi_1 = \frac{2C\gamma_{\text{cav}}(\delta + i)}{1 + \delta^2 + 2I}. \quad (14)$$

Φ_1 now includes a contribution due to the linear absorption and dispersion of the atoms. The contributions Φ_L and Φ_{NL} introduced below are simply the first two real terms of the expansion of formula (11) in powers of I . The total phase shift, including optical pumping now writes

$$\Phi_{\text{cav}} = \Phi_0 + \Phi_1(1 + p). \quad (15)$$

For consistency, the saturation of the optical pumping was also taken into account:

$$dp/dt = -\gamma_p p + \beta \frac{I}{1 + \delta^2 + 2I} (1 - p). \quad (16)$$

The evolution of the system was calculated again using this more elaborate model. This yielded only minor changes in the results, the general behaviour predicted by the simple model for the instabilities remaining the same.

Let us note that for high powers, the probe laser tends to push the atoms out of the beam, especially when its frequency is close to resonance. This phenomenon could give rise to bistability due to the change of the effective linear index of refraction of the medium with the number

of atoms in the interaction zone. A careful study of the behaviour of the cold atoms in the probe beam has shown that such mechanical effects were negligible under our experimental conditions.

IV. CONCLUSION

The recent development of the magneto-optic trap, which enables to get clouds of motionless atoms with a density comparable to that of the atomic beams is of great interest for nonlinear optics. In such traps, one can get large non-linearities by setting the driving field close to resonance without having much absorption. The instabilities described and interpreted in this paper show that it is possible to get new nonlinear phenomena in well characterized conditions. These experiments open the way to nonlinear and quantum optics using cold atoms in resonant cavities.

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